3

dpa/Corbis

Time Is Money

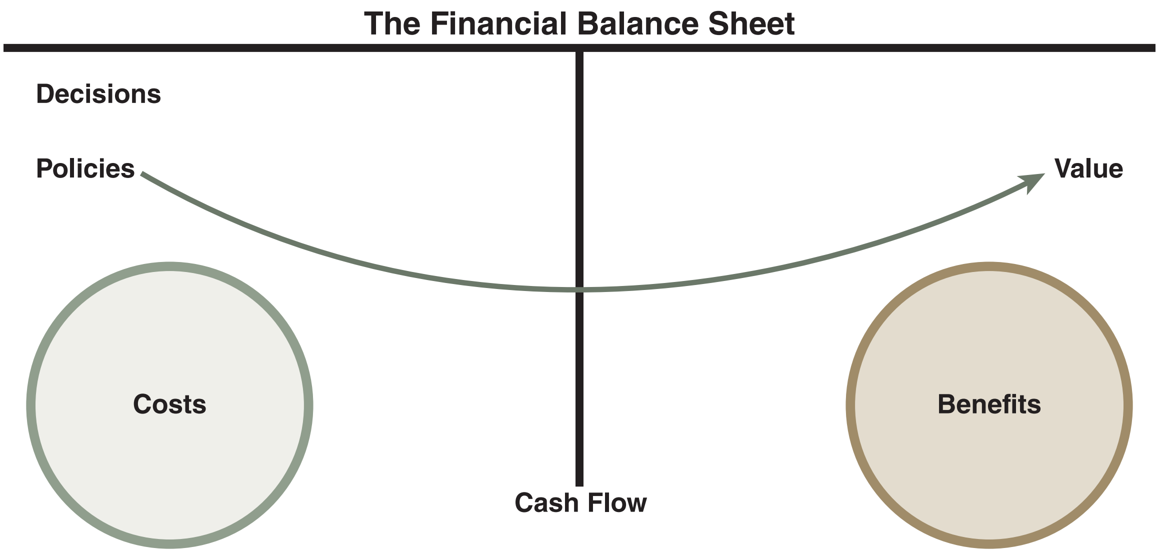
Learning Objectives

After studying this chapter, you should be able to:

- Express the time value of money and related mathematics, including present and future values, principal, and interest.
- Explain the significance of compounding frequency in relation to future and present cash flows and effective annual percentage rates.
- Identify the values of common cash flow streams, including perpetuities, annuities, and amortized loans.

Introduction

Figure 3.0: Chapter 3 in focus



The value of the firm depends on the size, timing, and riskiness of cash flows. Chapter 3 develops the mathematical techniques for valuing cash flows that occur at different points of time.

The saying "time is money" could not be more true than it is in finance. People rationally prefer to collect money earlier rather than later. By delaying the receipt of cash, individuals forgo the opportunity to purchase desired goods or invest the funds to increase their wealth. The foregone interest, which could be earned if cash were received immediately, is called the **opportunity cost** of delaying its receipt. Individuals require compensation to reimburse them for the opportunity cost of not having the funds available for immediate investment purposes. This chapter describes how such opportunity costs are calculated. Because many business activities require computing a value today for a series of future cash flows, the techniques presented in this chapter apply not only to finance but also to marketing, manufacturing, and management. Here are examples of questions that the tools introduced in this chapter can help answer:

- How much should we spend on an advertising campaign today if it will increase sales by 5% in the future?
- Is it worth buying a new computerized lathe for $120,000 if the lathe reduces material waste by 15%?
- Which strategy should we employ, given their respective costs and estimated contributions to future earnings?
- What types of health insurance and retirement plans are best for our employees, given the amount of money we have available?

Being able to value the cash to be received in the future—whether dividends from a share of stock, interest from a bond, or profits from a new product—is one of the primary skills needed to run a successful business. This chapter provides you with an introduction to that skill.

Pre-Test

1. Compound interest has no impact on the time value of money formula.
   a. True
   b. False

2. All else being equal, the higher the discount rate is, the smaller the present value will be.
   a. True
   b. False

3. An annuity pertains only to payments made.
   a. True
   b. False

Answers
1. **b.** False. *The answer can be found in Section 3.1.*
2. **a.** True. *The answer can be found in Section 3.2.*
3. **b.** False. *The answer can be found in Section 3.3.*

3.1 The Time Value of Money

The **time value of money** and the mathematics associated with it provide important tools for comparing the relative values of cash flows received at different times. Just as a hammer may be the most useful item in a carpenter's toolbox, time value of money mathematics is indispensable to a financial manager.

Recall from Chapter 1 that to increase shareholder wealth managers must make investments that have greater value than their costs. Often, such investments require an immediate cash outlay, like buying a new delivery truck. The investment (the truck) then produces cash flows for the corporation in the future (delivery fee income, increased sales, lower delivery costs, etc.). To determine whether the future cash flows have greater value than the initial cost of the truck, managers must be able to calculate the present value of the future stream of cash flows produced by this investment. Let's take a closer look at the time value of money in action.

Present and Future Value

Suppose a friend owes you $100 and the payment is due today. You receive a phone call from this friend, who says she would like to delay paying you for one year. You may reasonably demand a higher future payment, but how much more should you receive? The situation is illustrated in Figure 3.1 as a timeline.

Figure 3.1: Determining future value

How do we determine the future value of $100 today?

In this diagram "now," the present time, is assigned $t = 0$, or time zero. One year from now is assigned $t = 1$. The **present value** of the cash payment is $100 and is denoted PV_0. Its **future value** at $t = 1$ is denoted as FV_1. To find the amount that you could demand for deferring receipt of the money by one year, you must solve for FV_1, the future value of $100 one year from now. The FV_1 value will depend on the opportunity cost of forgoing immediate receipt of $100. You know, for instance, that if you had the money today you could deposit the $100 in a bank account earning 5% interest annually. However, you know from Chapter 2 that value depends on risk. In your judgment, your friend is less likely to pay you next year than is the bank. Therefore, you will increase the rate of interest to reflect the additional risk that you think is inherent in the loan to your friend.

Suppose that you decide that a 10% annual rate of interest is appropriate. The amount of the future payment, FV_1, will be the original **principal** plus the **interest** that could be earned at the 10% annual rate. Algebraically, you can solve for FV_1, being careful always to convert percentages to decimals when doing arithmetic calculations,

$$FV_1 = \$100 + \$100\ (0.10)$$

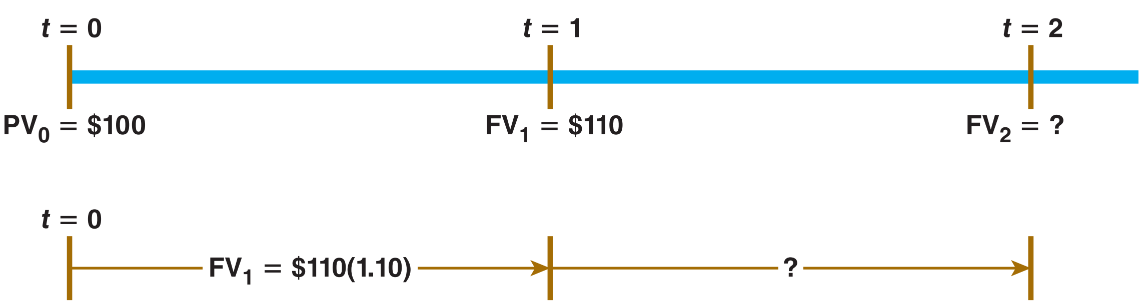
Factoring $100 from the right-hand side of the equation, we have

$$FV_1 = \$100(1 + 0.10) = \$100(1.10) = \$110$$

You may demand a $110 payment at $t = 1$ in lieu of an immediate $100 payment because these two amounts have equivalent *value*. Notice that if you had deposited the $100 in the bank, you would have only $105 after one year. The higher the interest rate, the faster the amount will grow.

Let's say that your friend agrees to this interest rate but asks to delay payment for two years. The new scenario is illustrated in Figure 3.2.

Figure 3.2: Determining the future value of $100 at %10 interest



How do we determine the future value of $100 today at a 10% interest rate?

Now we must find FV_2, the future value of the payment 2 years from today. Since we know $FV_1 = \$110$ and we know the interest rate is 10%, we can solve for FV_2 by recognizing that FV_2 will equal FV_1 plus the interest that could be earned on FV_1 during the second year.

$$FV_2 = FV_1 + FV_1(0.10) = \$110 + (\$110)(0.10) = \$110(1 + 0.10) = \$121$$

You may demand a \$121 payment at $t = 2$ because its time value is equivalent to either \$110 at $t = 1$ or \$100 at $t = 0$, given the 10% interest rate.

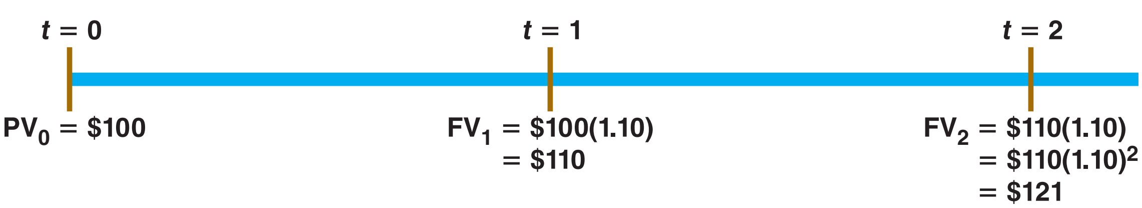
Even when lending money to a friend, it's important to iron out the details, agree upon the terms of repayment, and figure out the time value of the money.

The Agency Collection/Getty Images

Simple and Compound Interest

We just showed that, at a 10% annual interest rate, \$100 today is equivalent to \$110 a year from now and \$121 in 2 years. Now, we look at how compound and simple interest affect the time value of money. Look at the timeline shown in Figure 3.3.

Figure 3.3: The future value of \$100 at 10% compound interest



How does 10% compound interest impact the value of \$100 after two years?

This result may be generalized using the following formulas,

$$FV_1 = PV_0(1 + r) \tag{3.1}$$

$$FV_2 = PV_0(1 + r)^2 \tag{3.2}$$

where r is the interest rate.

Equation (3.2) can be restated as

$$\text{(3.3)} \qquad FV_2 = PV_0(1 + 2r) + PV_0(r^2)$$

Equation (3.3) is broken down in a special way. The first term on the right side of the equal sign, PV_0 $(1 + 2r)$, would yield \$120 given the information we have used in our example. The second term, PV_0 (), yields \$1. The value \$120 equals your original principal (\$100) plus the amount of interest earned (\$20) if your friend paid **simple interest**. Simple interest means that the same dollar amount of interest is received every period. For example, if you withdraw interest earned during each year at the end of that year, you would earn simple interest. In this case, you would receive \$10 interest payments at the end of years 1 and 2, totaling \$20. If, on the other hand, your friend credited (but did not pay) interest to you every year, then you would earn interest during year 2 on the interest credited to you at the end of year 1. Earning interest on previously earned interest is known as **compounding**. Thus, you would earn an extra dollar, a total of \$121, over the two-year period with interest *compounded annually*. The example assumed annual compounding since nearly all transactions are now based on compound rather than simple interest. This problem is demonstrated in the *Applying Finance: Annual Compounding* feature. See Appendix A for information on setting up your calculator, and additional financial application problems.

Applying Finance: Annual Compounding

Future Value With Annual Compounding: To solve the problem we just looked at with a financial calculator or Excel is straightforward.

To Solve Using TI Business Analyst

After clearing the calculator, use the following inputs:

100 [+/−]	[PV]
2	[N]
0	[PMT]
10	[I/Y]
[CPT]	[FV]

= \$121.00

Note: These may be input in any order so long as the FV and Compute are at the end. We entered 100 as a negative number. Think of it as \$100 going away (you are giving it to the bank) and the \$121 is being received (the bank is giving it back to you), so the two cash flows will have opposite signs. If you enter 100 [+/–] [PV] in this problem, then your answer will be a positive 121. If you entered the PV of \$100 as a positive number, then the FV displayed would be signed negative.

To Solve Using Excel

Use the FV function. The inputs for this function are: FV(RATE,NPER,PMT,PV,TYPE)

RATE: Interest rate per period as a %

NPER: Number of compounding periods

PMT: Any periodic payment (for the FV of a single cash flow this would be zero)

(***continued***)

Applying Finance: Annual Compounding (***continued***)

PV: Present value

TYPE: 0 if payments are made at the end of the period (most common) and 1 if payments are made at the beginning of the period

=FV(10%,2,0,-100,0)

=$121.00

Note: Financial functions in Excel require that cash inflows and cash outflows have different arithmetic signs. We signed the PV (the amount you put in the bank today) negative because it is flowing away from you and into the bank. The result ($121.00) is positive because that is a cash flow to you. Commas separate the inputs, so you cannot enter numbers with commas separating thousands (e.g., $1,000). Nor can you include dollar signs ($).

Let's extend this example to 20 years to better show the difference between simple and compound interest. At a 10% interest rate using simple interest our original deposit of $100 would grow into $300 over 20 years. This growth is based on receiving $10 of interest each year.

$$\$100 + (20 \times \$10) = \$100 + \$200 = \$300$$

Compare this to the result from compound interest.

$$FV_{20} = PV_0(1 + r)^{20} = 100(1 + 0.10)^{20} = \$672.75$$

The difference between simple and compound interest is $372.75 over 20 years!

Not all compounding is done on an annual basis, however. Sometimes interest is added to an account every six months (semiannual compounding). Other contracts call for quarterly, monthly, or daily compounding. As you will see in the next section, the frequency of compounding can make a big difference when the time value of money is calculated.

Many of the contracts on the bills we pay involve compound interest, such as with our credit cards. Can you think of other examples of where compound interest is utilized?

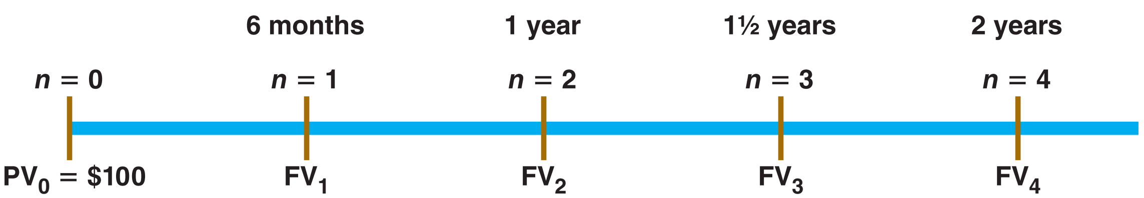
Rob Lewine/Getty Images

3.2 Valuing a Single Cash Flow

As mentioned in the previous section, how often a loan's interest is compounded changes how we determine the time value of money. There are different compounding periods: mortgage or car loans use monthly compounding; corporate bonds that pay interest semiannually use semiannual compounding; some certificates of deposit use continuous compounding; and many credit cards use daily compounding. In this section, we will show how different compounding periods affect the time value of money formula used.

Continuing our example from Section 3.1, let us suppose that your friend who wishes to delay paying you agrees to a 10% annual rate of interest over the two-year period and will allow you to compound interest semiannually. What will you be paid in two years given this agreement? *Semiannual compounding* means that interest will be credited to you every six months, based on half of the annual rate. In effect you will be earning a 5% semiannual rate of interest over four six-month periods. In other words, the **periodic interest rate** will be half the annual rate because you are using semiannual compounding *and* you will be earning interest for four *time periods* (n = 1 through 4), each period being one-half year long. The new situation is illustrated in Figure 3.4.

Figure 3.4: Semiannual compounding



How does semiannual interest impact the value of \$100 after two years?

Here, FV_1 is the future value of the \$100 at the end of period 1 (the first six months). As before, FV_1 equals the \$100 beginning principal plus interest earned over the six months at the 5% semiannual interest rate.

$$FV_1 = \$100 + \$100(0.05) = \$100(1 + 0.05) = \$105$$

Therefore, at the end of period 1 (at n = 1), the principal balance you are owed will be \$105. FV_2 will be equal to the principal at the beginning of period 2 plus interest earned during period 2.

$$FV_2 = \$105 + \$105(0.05) = \$110.25$$

Note that we could substitute [\$100(1.05)] for \$105 in the previous equation. Doing so, FV_2 could be expressed as follows:

$$FV_2 = \$105(1.05) = [\$100(1.05)](1.05) = \$100(1.05)^2$$

Following this pattern, finding FV_3 and FV_4 is straightforward.

$$FV_3 = \$100(1.05)^3 = \$115.76$$

$$FV_4 = \$100(1.05)^4 = \$121.55$$

The final equation gives the answer we seek. The future value at the end of four six-month periods is \$121.55. Changing from annual compounding to semiannual compounding has increased the future value of your friend's obligation to you by \$0.55. The additional interest earned from semiannual compounding, \$0.55, doesn't seem like much but imagine a firm borrowing \$100 million; then the amount earned from compounding—the interest earned on previous interest—can turn into tens of thousands of dollars.

The Future Value of a Single Cash Flow

The pattern established here may be generalized into the formula for the future value of a single cash flow using compound interest.

$$FV_n = PV_0(1 + r)^n \tag{3.4}$$

where

FV_n = the future value at the end of n time periods

PV_0 = the present value of the cash flow

r = the periodic interest rate

which equals the *annual nominal rate* divided by the number of compounding periods per year,

$$r = \frac{\text{Annual nominal rate}}{\text{Number of periods per year}}$$

n = the number of compounding periods until maturity, or

n = (number of years until maturity)(compounding periods per year)

It is critical when using this formula to be certain that r and n agree with each other. If, for example, you are finding the future value of \$100 after six years and the annual rate is 18%, compounded monthly, then the appropriate r is 1.5% per month (18%/12 = 1.5%) and n is 72 months (6 years times 12 months per year = 72 months). Students often adjust the interest rate and then forget to adjust the number of periods (or vice versa)! The answer to this problem is

$$FV_{6x12} = \left(1 + \frac{0.18}{12}\right)^{6x12}$$

$$FV_{72} = \$100(1.015)^{72} = \$292.12$$

For *quarterly compounding*, you would divide the annual rate by 4 and multiply the number of years by 4. So \$100 after six years and an annual rate of 18% with quarterly compounding would be found using a periodic interest rate of 4.5% (18%/4) and 24 periods (6 years times 4 periods per year).

$$\mathrm{FV}_{6x4} = \left(1 + \frac{0.18}{4}\right)^{6x4}$$

$$\mathrm{FV}_{24} = \$100(1.045)^{24} = \$287.60$$

For simple interest, without compounding, the future value is simply equal to the annual interest earned, times the number of years, plus the original principal. The formula for the *future value of a single cash flow using simple interest* is

$$(3.5) \qquad \mathrm{FV}_n^s = \mathrm{PV}_0 + (n)(\mathrm{PV}_0)(r) = \mathrm{PV}_0(1 + nr)$$

where

FV_n^s = the future value at the end of n periods using simple interest

n = the number of periods until maturity (Generally n simply equals the number of years because there is no adjustment for compounding periods.)

r = the periodic rate (which also usually equals the annual rate because there is no adjustment for compounding periods)

For the previous example, the future value of \$100 invested for 6 years in an account paying 18% per year using simple interest is

$$\mathrm{FV}_6^s = \$100[1 + (6)(0.18)] = \$208.00$$

The adjustment process we just discussed works for all compounding periods except one: *continuous compounding*. We won't go into the details of the math; we will just show the result.

$$(3.6) \qquad \mathrm{FV}_n = \mathrm{PV}_0(e^{rn})$$

The letter e is one of those special numbers in mathematics that is assigned its own name. (Another one is π, which you may remember is approximately equal to 3.14.) The number e is approximately equal to 2.72 (more precisely 2.71828183). The exponent rn in Equation (3.6) doesn't need to be adjusted. In our example of 18% for six years, rn will be the same with or without any of those adjustments we discussed. We always do math with decimals rather than percentages, so the rn exponent for 18% would be $0.18 \times 6 = 1.08$ as is 1.5% (0.015×72) or 4.5% (0.045×24). Most calculators have an e key, which makes computing continuous compounding fairly easy.

Monthly compounding yielded a future value after six years of \$292.12, or \$84.12 more than simple interest in this example. Table 3.1 illustrates the future value of \$100, bearing 18% annual interest, with different compounding assumptions. Use your calculator to replicate the solutions illustrated below. Be sure your n and r agree (e.g., both are monthly, yearly, etc.), and always be sure you express percentages as decimals before doing any calculations. You should practice with your calculator until your answers match those given. For more applications, refer to the *Applying Finance: Future Value* feature box.

Table 3.1: The future value of $100 after six years at 18% annual interest, various compounding periods

Compounding assumption	*n*	*r*	FV_n
Annual	6	0.18	$269.96
Semiannual	12	0.09	$281.27
Quarterly	24	0.045	$287.60
Monthly	72	0.015	$292.12
Weekly	312	0.00346	$293.92
Daily	2,190	0.000493	$294.39
Continuous	∞		$294.47

Applying Finance: Future Value

Future Value of Single Cash Flow: If you put $400 in the bank today at 12% per year, leave it there for five years, what will be the balance at the end of the time period?

To Solve Using TI Business Analyst

400 [PV]

5 [N]

0 [PMT]

12 [I/Y]

[CPT] [FV]

= $704.9366

Note: These may be input in any order so long as the FV and Compute are at the end. Also, the calculator register will show the answer as a negative 704.9366 since you entered 400 as a positive number. Think of it as 400 is cash going one way (you are giving it to the bank) and the 704 is going the opposite direction (the bank is giving it back to you), so the two cash flows will have opposite signs. If you enter 400 [+/–] [PV] in this problem, then your answer will be a positive 704.9366. It does not matter which way you do this.

To Solve Using Excel

Use the FV function. The inputs for this function are: FV(RATE,NPER,PMT,PV,TYPE)

RATE: Interest rate per period as a %

NPER: Number of compounding periods

PMT: Any periodic payment (for the FV of a single cash flow this would be zero)

PV: Present value

TYPE: 0 if payments are made at the end of the period (most common) and 1 if payments are made at the beginning of the period

(***continued***)

Applying Finance: Future Value (***continued***)

If you put $400 in the bank today at 12% per year, leave it there for five years, what will be the balance at the end of the time period?

=FV(12%,5,0,-400,0)

= $704.94

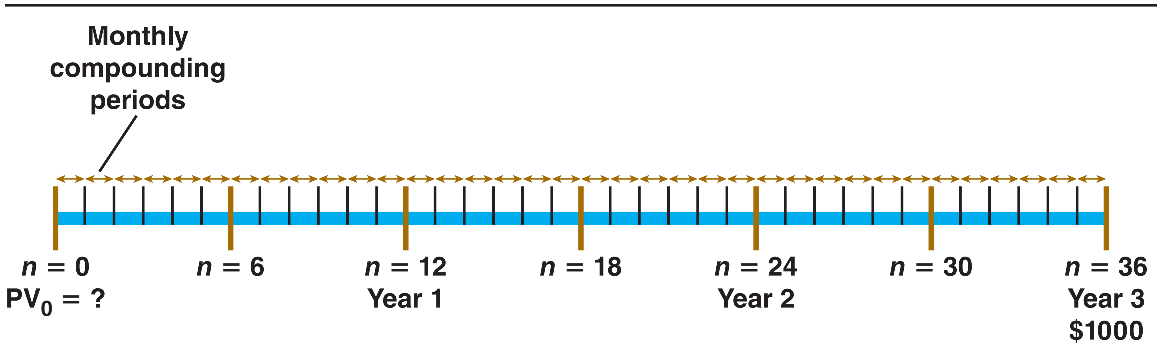
Note: Financial functions in Excel require that cash inflows and cash outflows have different arithmetic signs. We signed the PV (the amount you put in the bank today negative because it is flowing away from you and into the bank. The result ($704.94) is positive because that is a cash flow to you. The inputs are separated by commas, so you cannot enter numbers with commas separating thousands (e.g., $1,000). Nor can you include dollar signs ($).

The Present Value of a Single Cash Flow

We have solved for the future value of a current cash flow. Often, we must solve for the *present value of a future cash flow*, solving for PV rather than FV. You can think of the present value as the amount that you have to put in the bank today to have some specific amount in the future. A higher interest rate causes a deposit to grow faster, so the higher the interest rate the smaller the amount of money that has to be deposited today to achieve a desired future amount. Similarly, the longer the time until a future cash flow is collected, the smaller the amount deposited has to be. This is because the initial deposit has more time to grow.

Suppose, for example, you are going to receive a bonus of $1,000 in three years. You could really use some cash today and are able to borrow from a bank that would charge you an annual interest rate of 12%, compounded monthly. You decide to borrow as much as you can now such that you will still be able to pay off the loan in three years using the $1,000 bonus. In essence, you wish to solve for the present value of a $1,000 future value, knowing the interest rate (12% per year, compounded monthly) and the term of the loan (3 years, or 36 monthly compounding periods). Figure 3.5 is a timeline illustrating the problem. This problem is also practiced in the *Applying Finance: Present Value* feature.

Figure 3.5: Determining present value of $1000 in the future



How much can you borrow today at 12% compounded monthly for 36 months, if you plan to pay it back with your $1000 bonus at the end of the loan?

Applying Finance: Present Value

Present Value of Single Cash Flow: How much money would you have to put in the bank today at 12% per year, with monthly compounding, to have $1,000 in exactly three years?

To Solve Using TI Business Analyst

Clear TVM worksheet

2nd [CLR TVM]

2nd [Quit]

Clear CF worksheet

2nd [CLR WORK]

2nd [Quit]

Set the compounding period to monthly

2nd [P/Y] 12 [enter]

2nd [Quit]

1000 [FV]

36 [N]

0 [PMT]

12 [I/Y]

[CPT] [PV]

= $698.92

To Solve Using Excel

Use the PV function with the format: PV(RATE,NPER,PMT,FV,TYPE).

The inputs for this example would be:

=PV(1%,36,0,1000,0)

= –$698.92

In this case $n = 36$, $r = 1\%$, and is known, whereas PV_0 is unknown. We may still use Equation (3.4), substituting in the known quantities and using some algebra.

$$\text{(3.4)} \qquad FV_n = PV_0(1 + r)^n$$

$$\$1{,}000 = PV_0(1.01)^{36}$$

$$PV_0 = \$1{,}000(1.01)^{-36} = \$1{,}000\frac{1}{1.01^{36}} = \$698.92$$

You could borrow $698.92 today and fully pay off the loan, given the bank's terms, in three years using your $1,000 bonus. We can generalize the last expression into the *formula for the present value of a single cash flow with compound interest*. Solving for the present value of a future cash flow is also known as **discounting**. In fact, compounding and discounting are flip sides of the same coin. Compounding is used to express a value at a future date given a rate of interest. Discounting involves expressing a future value as an equivalent amount at an earlier date.

This formula is also called the *discounting formula for a single future cash flow*.

$$\text{(3.7)} \qquad PV_0 = FV_n(1 + r)^{-n} = FV_n\frac{1}{(1 + r)^n}$$

The variables PV_0, FV_n, n, and r are defined exactly as they are in the future value formula because both formulas are really the same, just solved for different unknowns.

To find the present value with continuous compounding, we would use

$$\text{(3.8)} \qquad PV_0 = FV_n(e^{-rn}) = FV_n\frac{1}{e^{rn}}$$

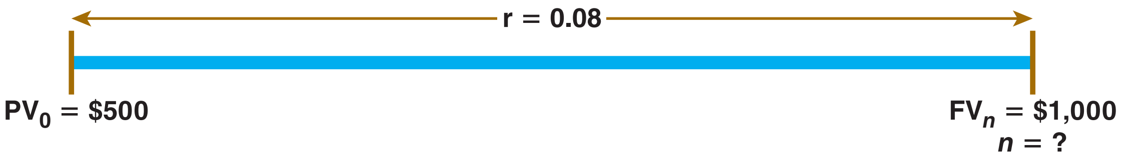
Table 3.2 solves for the present, or discounted, value of a \$1,000 cash flow to be received in 1 year at a 12% per year **discount rate** using different compounding periods. You should be able to replicate these solutions on your calculator.

Table 3.2: The present value of \$1,000 to be received in 1 year discounted at 12% annual interest, various compounding periods

Compounding assumption	*N*	*R*	PV_N
Annual	6	0.12	\$892.86
Semiannual	12	0.06	\$890.00
Quarterly	24	0.03	\$888.49
Monthly	72	0.01	\$887.45
Weekly	312	0.00231	\$887.04
Daily	2,190	0.000329	\$886.94
Continuous	∞		\$886.92

Present and future value formulas are very useful because they may be used to solve a variety of problems. Suppose you make a \$500 deposit in a bank today and you want to know how long it will take your account to double in value, assuming that the bank pays 8% interest per year, compounded annually (shown in Figure 3.6). Here, you are solving for the number of time periods. You may substitute the known quantities $PV_0 = \$500$, $FV_n = \$1,000$, $r = 0.08$ into either formula and solve for n:

Figure 3.6: Determining number of time periods



How many time periods must pass to double your investment of $500 at 8% interest compounded annually?

(3.7) $$PV_0 = FV_n(1 + r)^{-n}$$

$$\$500 = \$1{,}000(1.08)^{-n}$$

$$(1.08)^{n} = \$1{,}000/\$500$$

$$(1.08)^{n} = 2$$

At this point, without using logarithms, you must use trial and error to solve for n. Suppose you try n =10 as your first guess for n:

$$(1.08)^{10} = 2.1589$$

This value yields a number higher than our objective of 2. Therefore, try $n = 9$ because a lower value of n will yield a lower answer:

$$(1.08)^{9} = 1.999$$

which is close enough. In nine years, the balance in your account will double.

Suppose the account earned 8% per year compounded monthly. To find the time until the account's balance doubled, you would convert the interest rate to reflect monthly compounding $r = \frac{0.08}{12} = 0.00667$ and solve for the number of compounding periods.

(3.7) $$PV_0 = FV_n(1 + r)^{-n}$$

$$\$500 = \$1{,}000(1.00667)^{-n}$$

$$(1.00667)^{n} = 2$$

Using trial and error, you get the answer $n = 105$. This should be interpreted as 105 months because you are dealing with monthly compounding periods. Thus, in 8.75 years the account will double in value when using monthly rather than annual compounding.

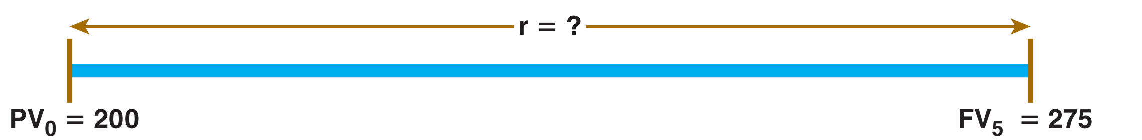
This example illustrates an important lesson. It takes less time to achieve a desired amount of wealth with more frequent compounding at a given nominal interest rate. It is no surprise that borrowers prefer less frequent compounding, while lenders prefer compounding as frequently as possible. The difference between compounding frequencies offered at various banks makes shopping around worthwhile whether you are a borrower or a saver.

Finding Interest Rates

Another type of problem is solving for the interest rate. This time let's suppose that an investment costing \$200 will make a single payment of \$275 in 5 years. What is the interest rate such an investment will yield? Substitute $n = 5$ into the formula and solve for r.

Figure 3.7 shows a timeline for finding the interest rate that equates a \$200 deposit to a future value of \$275 in 5 years.

Figure 3.7: Determining interest rate



What is the interest rate on an investment of \$200 that results in a single payment of \$275 in five years?

We can use Equation (3.7) to find r.

$$\text{(3.7)} \qquad PV_0 = FV_n(1 + r)^{-n}$$

$$\$200 = \$275(1 + r)^{-n}$$

$$(1 + r)^5 = \frac{\$275}{\$200}$$

$$(1 + r)^5 = 1.375$$

$$1 + r = (1.375)^{1/5}$$

$$1 + r = 1.375^{0.20}$$

$$r = (1.375)^{0.20} - 1$$

$$r = 0.06576$$

The answer, $r = 0.06576$, is based on an annual compound rate because we assumed $n = 5$ years. It is also expressed as a decimal and could be re-expressed as a percentage, 6.576% per year compounded annually. See the *Applying Finance: Finding Interest Rates* feature for practice with this problem.

Applying Finance: Finding Interest Rates

Annual Compound Interest Rate: If a $200 deposit grows into $275 in five years, what is the annual compound interest rate?

To Solve Using TI Business Analyst

(Make sure to set P/Y to 1).

275	[FV]
5	[N]
0	[PMT]
200 [+/–]	[PV]
[CPT]	[I/Y]

= 6.58

To Solve Using Excel

Use the Rate function with the format: PV(NPER,PMT,PV,FV,TYPE,GUESS).

The inputs for this example would be:

= RATE(5,0,-200,275,,)

= 6.5763%

Effective Annual Percentage Rate

As you have seen, the frequency of compounding is important. Truth-in-lending laws now require that financial institutions reveal the **effective annual percentage rate (EAR)** to customers so that the true cost of borrowing is explicitly stated. Before this legislation, banks could quote customers annual interest rates without revealing the compounding period. Such a lack of disclosure can be costly to borrowers. For example, borrowing at a 12% yearly rate from bank A may be more costly than borrowing from bank B, which charges 12.1% yearly, if bank A compounds interest daily and bank B compounds semiannually. Both 12% and 12.1% are **nominal rates**—they reveal the rate "in name only" but not in terms of the true economic cost. To find the effective annual rate, divide the **nominal annual percentage rate (APR)** by the number of compounding periods per year and add 1; then raise this sum to an exponent equal to the number of compounding periods per year. Finally, subtract 1 from this result.

$$\text{(3.9)} \qquad \text{EAR} = \left(1 + \frac{\text{APR}}{\text{CP}}\right)^{cp} - 1$$

For our example,

$$\text{EAR}_A = \left(1 + \frac{0.12}{365}\right)^{365} - 1 = 0.1275 = 12.75\%$$

$$\text{EAR}_B = \left(1 + \frac{0.121}{2}\right)^{2} - 0.1247 = 12.47\%$$

Thus, if you are a borrower, you would prefer to borrow from bank B despite its higher APR. The lower EAR translates into a lower cost over the life of the loan. The disclosure of EARs makes comparison shopping for rates much easier.

3.3 Valuing Multiple Cash Flows

Many problems in finance involve finding the time value of multiple cash flows. Consider the following problem. A charity has the opportunity to purchase a used mobile hot dog stand being sold at an auction. The charity would use the hot dog stand to raise money at special events held in the summer each year (at the county fair, baseball and soccer games, etc.). The old hot dog stands will only last two years and then will be worthless. The charity estimates that, after all operating expenses, the stand will produce cash flows of $1,000 in both June and July in each of the next two years and cash flows of $1,500 in each of the next two Augusts. The auction takes place January 1, and the charity requires that its fundraising projects return 12% on their invested funds. How much should the charity bid for the hot dog stand? The strategy for solving this problem is shown in Figure 3.8.

Figure 3.8: Determining the present value of multiple cash flows

$N = 0$	$N = 6$	$N = 7$	$N = 8$	$N = 18$	$N = 19$	$N = 20$
	June	July	August	June	July	August
	$1,000	$1,000	$1,500	$1,000	$1,000	$1,500

$$\$942.05 = PV_0 = \$1{,}000(1.01)^{-6} \leftarrow$$
$$\$932.72 = PV_0 = \$1{,}000(1.01)^{-7} \leftarrow$$
$$\$1{,}385.22 = PV_0 = \$1{,}500(1.01)^{-8} \leftarrow$$
$$\$836.02 = PV_0 = \$1{,}000(1.01)^{-18} \leftarrow$$
$$\$827.74 = PV_0 = \$1{,}000(1.01)^{-19} \leftarrow$$
$$\underline{\$1{,}229.32} = PV_0 = \$1{,}500(1.01)^{-20} \leftarrow$$
$$\$6{,}153.07 = \text{Total Present Value}$$

The charity should bid no more than $6,153.07 for the hot dog stand, given the stream of expected cash flows.

The present value of the stream of cash flows the stand is expected to produce is found by applying Equation (3.7) to each of the six future cash flows. Note that 1% is used as the periodic rate (12% per year/12 months) because cash flows are spaced in monthly intervals. The charity should bid a maximum of $6,153.07 for the hot dog stand. Given the level of expected cash flows, paying more than this amount would result in the charity earning a lower return than its 12% objective.

The hot dog stand example illustrates the general formula for finding the *present value of any cash flow stream*,

$$\text{(3.10)} \qquad PV_0 = \frac{CF_1}{(1+r)^1} + \frac{CF_2}{(1+r)^2} + \cdots + \frac{CF_N}{(1+r)^N} = \sum_{n=1}^{N} \frac{CF_n}{(1+r)^n}$$

where

n = the number of compounding periods from time 0

CF_n = the cash flow to be received exactly n compounding periods from time 0 (e.g., CF_1 is the cash flow received at the end of period 1, etc.)

r = the periodic interest rate

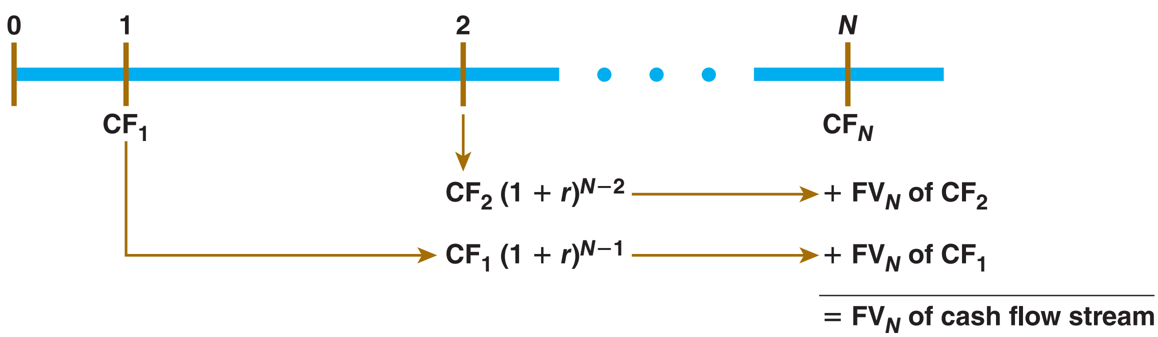
N = the number of periods until the last cash flow

The *future value formula for a cash flow stream* is also found by finding the future value of each individual cash flow and summing. Terms in the formula are defined as in the present value formula.

$$FV_N = CF_1(1 + r)^{N-1} + CF_2(1 + r)^{N-2} + \cdots + CF_N \qquad (3.11)$$

You may question why in Equation (3.11) the first term is raised to the exponent $N - 1$ and why the last term is not multiplied by an interest factor. This situation may be clarified by using a timeline, as shown in Figure 3.9. The last cash flow (CF_n) occurs at the end of the last time period and therefore earns no interest.

Figure 3.9: Determining the future value of cash flow streams



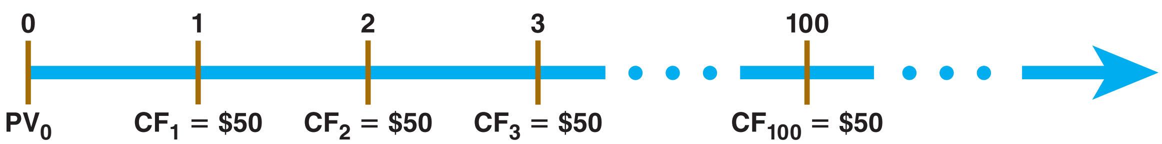
In this case, interest is deferred until after the first time period. This is not always true of future values of cash flow streams.

As the timeline shows, CF_1 will earn interest for $N - 1$ period, but CF_n earns no interest and is simply added to the other sums to find the total future value. By convention, we assume that the cash flows from investments do not start immediately but are deferred until the end of the first period. This is not always the case, however. Practitioners must carefully analyze any problem to be certain exactly when cash flows will occur. A timeline is a useful aid in modeling when the cash flows from a project will occur.

Perpetuities

Some special patterns of cash flows are frequently encountered in finance. The nature of these patterns allows the general formulas to be simplified to a more concise form. The first special case is that of **perpetuities**. These are cash flow streams where equal cash flow amounts are uniformly spaced in time (every year, every month, etc.). Perpetuity means that these payments continue forever. To illustrate, suppose an investment is expected to pay $50 every year forever. Investors require a return of 10% on this investment. What should be its current price? Recognizing that today's price should equal the present value of the investment's future cash flows, the problem is illustrated using a timeline in Figure 3.10.

Figure 3.10: Determining the present value of a perpetuity



What is the present value of a perpetuity that pays $50 a year forever, with a 10% return on investment?

The arrow indicates that these cash flows continue into the future indefinitely. This poses a problem: If there are an infinite number of cash flows, how can we find all of their present values? Let's consider the algebraic expression of this problem.

$$PV_0 = \frac{\$50}{(1.10)^1} + \frac{\$50}{(1.10)^2} + \cdots + \frac{\$50}{(1.10)^{100}} + \cdots$$

$$PV_0 = \$50\left[\frac{1}{(1.10)^1} + \frac{1}{(1.10)^2} + \cdots + \frac{1}{(1.10)^{100}} + \cdots\right]$$

Summing this geometric series and using some algebra yields the following formula for the *present value of a perpetuity*:

$$\text{(3.12)} \qquad PV_0 = \frac{CF}{r}$$

Note that there is no subscript attached to CF because all the cash flows are the same.

Therefore, there is no need to distinguish CF_1 from CF_2, and so on. Let's apply the formula to the example. CF = $50, $r = 0.10$, and

$$PV_0 = \frac{\$50}{0.10} = \$500$$

Annuities

Of all the special patterns of cash flow streams, *annuities* are the most common. As we shall see, millions of fixed-rate home mortgages are annuities. Retirement payments, bond interest payments, automobile loan payments, and lottery jackpot payoffs all often fit the annuity pattern.

An **annuity** is a stream of equally sized cash flows, equally spaced in time, which end after a fixed number of payments. Thus, annuities are like perpetuities, except they do not go on forever. The *present value of an annuity* can be found by summing the present values of all the individual cash flows.

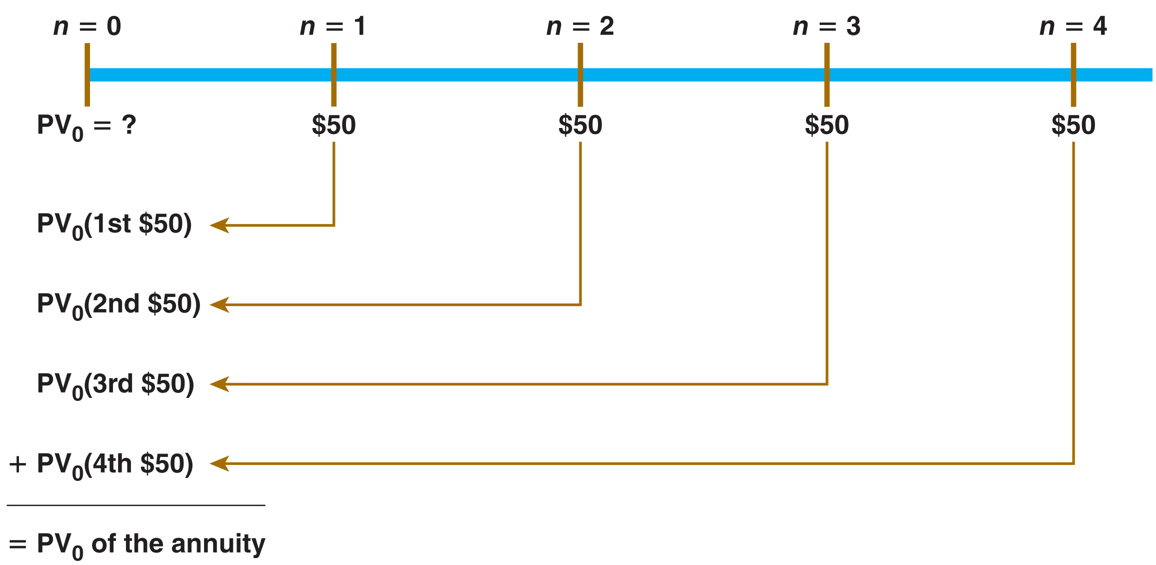
$$(3.10) \qquad PV_0 = \sum_{n=1}^{N} \frac{CF}{(1+r)^n} = \frac{CF}{(1+r)^1} + \frac{CF}{(1+r)^2} + \cdots + \frac{CF}{(1+r)^N}$$

Here N is the number of cash flows being paid and CF is the uniform amount of each cash flow. Solving for CF_0 using Equation (3.10) would be a time-consuming problem if n were large. However, because the right-hand side of the equation is yet another geometric series, it can be simplified to yield the formula for finding the *present value of an annuity*.

$$(3.13) \qquad PV_0 = (CF)\left(\frac{1 - [1/(1+r)^N]}{r}\right)$$

To convince you that Equations (3.10) and (3.13) are equivalent, let's work an example using both approaches. Suppose you wished to know the present value of a stream of \$50 payments made semiannually over the next two years. The first payment is scheduled to begin six months from today, and the annual rate of interest is 10%. The problem is illustrated with a timeline in Figure 3.11.

Figure 3.11: Determining the present value of an annuity



What is the present value of an annuity that pays \$50 semiannually for two years at 10% annual interest?

Using Equation (3.10), and recognizing that $r = 5\% = 0.05$ semiannually, this problem may be solved as follows:

$$PV_0 = \frac{\$50}{(1.05)^1} + \frac{\$50}{(1 + .05)^2} + \frac{\$50}{(1 + .05)^3} + \frac{\$50}{(1 + .05)^4}$$

$$PV_0 = \$177.30$$

Alternatively, Equation (3.13) could be used to solve the same problem.

$$PV_0 = (\$50)\frac{[1 - [1/(1.05)^4]}{0.05}$$

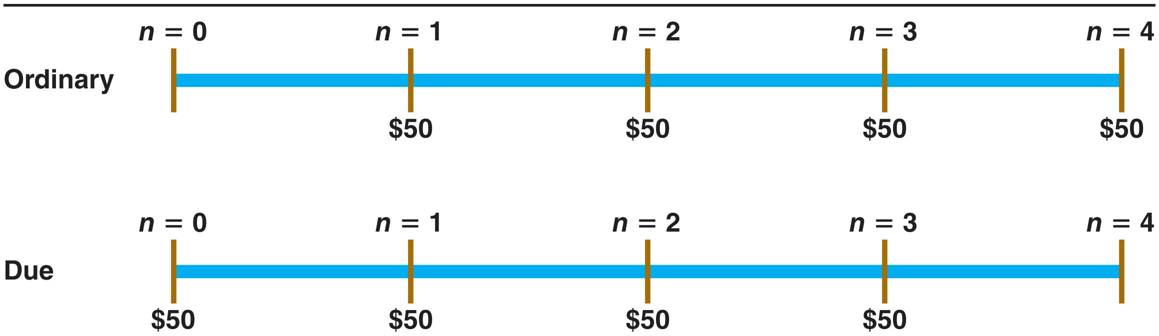
$$PV_0 = \$177.30$$

It may appear that using Equation (3.10) is just as time-consuming as using Equation (3.13), but consider the work involved had there been 300 payments rather than 4.

The problem just solved is an example of an **ordinary annuity** because cash flows commence at the end of the first period. Most loans require interest payments at the *end* of each period. Rent, on the other hand, is usually payable in advance. Annuities in which cash flows are made at the beginning of each period are called **annuities due**. Leases are usually structured as an annuity due; you make a payment before you get use of the asset. But loans are structured as regular annuities because some interest has to build up before a payment is made. Let's change the example we just worked slightly to require that the cash flows be made at the beginning of each period.

The timeline in Figure 3.12 shows that in a four-payment annuity due, each payment occurs one period sooner than in an otherwise similar ordinary annuity. Because of this characteristic, each cash flow is discounted for one less period when finding the PV of an annuity due.

Figure 3.12: An ordinary annuity versus an annuity due



Payments occur one period sooner in an annuity due, as opposed to an ordinary annuity.

The formula for finding the *present value of an annuity due* is

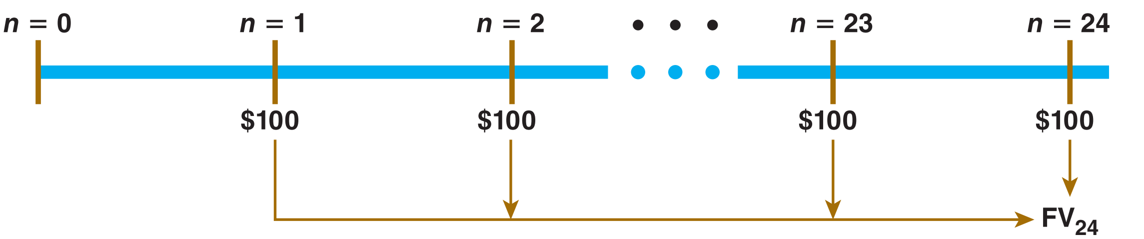
$$\text{(3.14)} \qquad PV_0^{due} = (CF)\left(\frac{1 - [1/(1 + r)^N]}{r}\right)(1 + r)$$

This is simply the formula for an ordinary annuity times $1 + r$, which adjusts for one less discounting period. Thus, it is usually easier to find the PV of an ordinary annuity and multiply times $1 + r$ when solving for the PV of an annuity due.

$$\text{(3.15)} \qquad PV_0^{due} = PV_0^{\text{ord}}(1 + r)$$

Now suppose you save $100 each month for two years in an account paying 12% interest annually, compounded monthly. What will be the balance in the account at the end of two years if you make your first deposit at the end of this month? Figure 3.13 illustrates this problem with a timeline.

Figure 3.13: Determining the future value of an ordinary annuity



What is the future value of an ordinary annuity that pays $100 a month for two years at 12% annual interest?

In this case we are trying to solve for the future value of an ordinary annuity.

$$FV_{24} = \$100(1.01)^{23} + 100(1.01)^{22} + \cdots + \$100$$

Solving our problem in this manner would take considerable time. Fortunately, the future value of an annuity is also a geometric series, which can be simplified.

The formula for the *future value of an ordinary annuity* is

$$\text{(3.16)} \qquad FV_N^{\text{ord}} = (CF)\frac{(1 + r)^N - 1}{r}$$

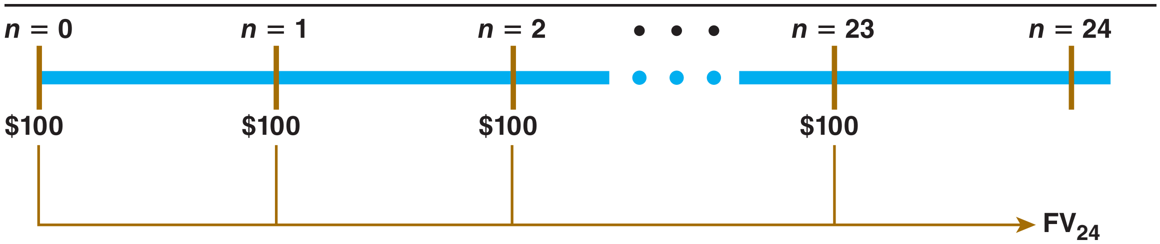
Substituting the values for our example into Equation (3.16) yields the solution

$$FV_{24} = (\$100)\frac{(1.01)^{24} - 1}{0.01}$$

$$FV_{24} = \$2{,}697.35$$

If the first deposit were made immediately, our problem would be one of finding the *future value of an annuity due*. Figure 3.14 illustrates this problem using a timeline.

Figure 3.14: Determining the future value of an annuity due



What is the future value of an annuity due that pays $100 a month for two years at 12% annual interest?

Each cash flow in an annuity due earns one additional period's interest compared to the future value of an ordinary annuity. Thus, the future value of an annuity due is equal to the future value of an ordinary annuity times $1 + r$.

$$\text{(3.17)} \qquad FV_N^{due} = FV_N^{ord}(1 + r)$$

$$FV_{24}^{due} = (\$2{,}697.35)(1.01) = \$2{,}724.32$$

The future value of the deposits would therefore increase to $2,724.32 if they were made at the beginning of each period. Notice the adjustment from an ordinary annuity to an annuity due is the same whether you are solving for PV or FV [compare Equations (3.15) and (3.17)]. Note that both the present value and the future value of an annuity due are always larger than an otherwise similar ordinary annuity.

Application: Loan Amortization

Many loans, such as home mortgages, require a series of equal payments made to the lender. Each payment is for an amount large enough to cover both the interest owed for the period as well as some principal. In the early stages of the loan, most of each payment covers interest owed by the borrower and very little is used to reduce the loan balance. Later in the loan's life, the small principal reductions have added up to a sum that has significantly reduced the amount owed. Thus, as time passes, less of each payment is applied toward interest and increasing amounts are paid on

Home mortgages are an important example of an amortized loan. Can you think of any other examples of this type of loan?

Getty Images News/Getty Images

the principal. This type of loan is called an **amortized loan**. The final payment just covers both the remaining principal balance and the interest owed on that principal. An amortized loan is a direct application of the present value of an annuity. The original amount borrowed is the present value of the annuity (PV_0), while loan payments are the annuity's cash flows (CFs).

If you borrow \$100,000 to buy a house, what will your monthly payments be on a 30-year mortgage if the interest rate is 9% per year? For this problem, the formula for finding the present value of an annuity is used [Equation (3.13]. The present value is the loan amount (PV_0 = \$100,000), there are 360 payments ($N = 360$), and the monthly interest rate is 0.75% (9%/12 months). The payment amount (CF) is determined as follows:

$$PV_0^{\text{ord}} = (\text{CF})\left(\frac{1 - [1/(1+r)^N]}{r}\right) \tag{3.13}$$

$$\$100{,}000 = \frac{(\text{CF})(1 - [1/(1.0075)^{360}]}{0.0075}$$

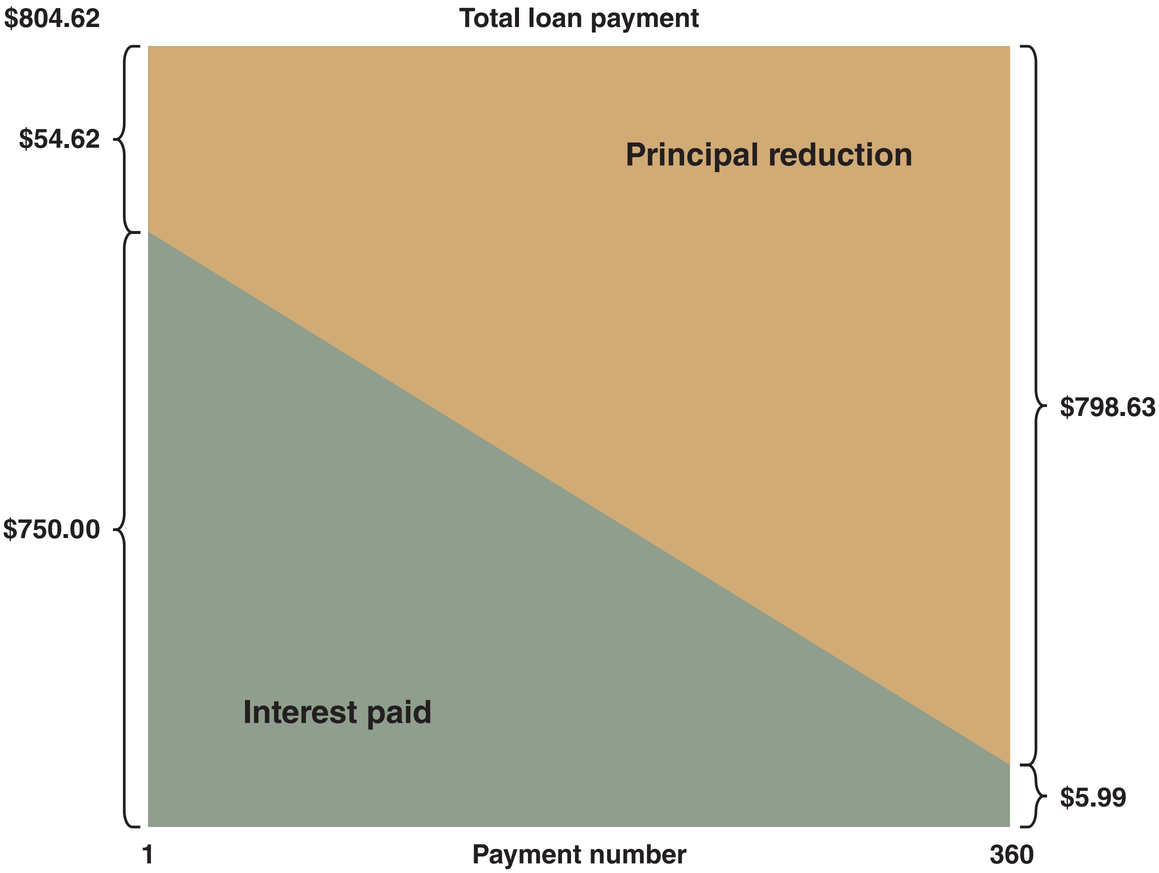
$$\$100{,}000 = \frac{(\text{CF})\left(1 - \dfrac{1}{14.730576}\right)}{0.0075}$$

$$\$100{,}000 = (\text{CF})\frac{(0.932114)}{0.0075} = (\text{CF})(124.2819)$$

$$(\text{CF})\frac{\$100{,}000}{124.2819} = \$804.62$$

A stream of 360 monthly payments of \$804.62 will cover the interest owed each month and will pay off the entire \$100,000 loan as well. Figure 3.15 illustrates how the amount of each payment applied toward principal increases over time, with a corresponding decrease in interest expense. As shown in Figure 3.15, \$750.00 of the first payment is used to pay the interest owed the lender for the use of \$100,000 during the first month at the 0.75% monthly rate. \$54.62 of the first payment will be applied toward the principal. Thus, for the second month of the loan only \$99,945.38 is owed. This reduces the amount of interest owed during the second month and increases the second month's principal reduction. This pattern continues until the last payment when, as seen in Figure 3.15, only a \$798.63 principal balance is remaining. The last month's interest on this balance is \$5.99. Therefore, the last \$804.62 payment will just pay off the loan and pay the last month's interest, too. Note that the ending balance (or the FV) of the loan equals zero after the last payment is made, so the loan is completely paid off with the last payment.

Figure 3.15: Components of an amortized loan over time



In an amortized loan—such as a typical home loan—the interest portion of the loan payment gets smaller over time, so more principal is repaid with each payment, and the amount paid each month remains the same: $804.62. After 30 years, the $100,000 loan will be paid off, plus the 9% annual interest.

Table 3.3 is an *amortization table* showing principal and interest payments on a five-year, $10,000 loan, amortized using a 10% rate compounded annually. An amortization table is useful because it can be used to find the unpaid balance owed on a loan after some payments have been made. Using Table 3.3, a borrower would know, for example, that $4,578.32 would be necessary to pay off the loan after the third annual payment is made.

Table 3.3: Loan amortization table for $10,000 borrowed at 10% interest annually compounded for five years

Year	Beginning principal balance	Total payment	Interest	Principal reduction	Ending principal balance
1	$10,000	$2,637.97	$1,000	$1,637.97	$8,362.03
2	$8,362.03	$2,637.97	$836.20	$1,803.77	$6,560.26
3	$6,560.26	$2,637.97	$656.03	$1,981.94	$4,578.32
4	$4,578.32	$2,637.97	$457.83	$2,180.14	$2,398.18
5	$2,398.18	$2,637.97	$239.79	$2,398.18	$0

Note: Each year's beginning balance equals the previous year's ending balance. Each year's interest equals the rate multiplied by the total loan amount. Each year's principal reduction equals the total payment minus the amount applied toward the interest. Each year's ending principal balance equals the beginning balance minus the principal reduction.

Field Trip: Loan Amortization

Bankrate.com provides a mortgage amortization schedule calculator that allows users to build an amortization table using their own data.

Visit: http://www.bankrate.com/calculators/mortgages/amortization-calculator.aspx

Experiment with different mortgage amounts, terms, and interest rates to see how they affect your monthly payments. You can even experiment with adding additional payments to change the pay-off date of the loan.

Conclusion

Chapter 3 has covered much of the topic of the time value of money. Next, the concepts and techniques introduced here will be applied to finding the value of stocks, bonds, and other securities. Before that, however, it is best to practice the newly acquired skills. The authors cannot overemphasize the importance of mastering time value mathematics. Therefore, as you do your homework, make sure you feel confident in your ability. If you are not, now is a good time to ask your instructor for assistance.

Post-Test

1. Compound interest has no impact on the time value of money formula.
   a. True
   b. False

2. All else being equal, the higher the discount rate is, the smaller the present value will be.
   a. True
   b. False

3. An annuity pertains only to payments made.
   a. True
   b. False

4. If you loan a friend $250 for one year, at a rate of 15% annual interest, what is the future value of that loan?
   a. $255.00
   b. $275.00
   c. $290.75
   d. $287.50

5. What is the present value of $1,000 to be received in one year with annual compounding at a 10% per year rate?
   a. $900.00
   b. $909.09
   c. $990.90
   d. $1,100

6. Suppose you are purchasing an automobile. You borrow $12,000 and make quarterly payments (four per year) on a four-year amortized loan. If the interest rate is 8% per year, what is your quarterly payment?
   a. $750.00
   b. $818.43
   c. $844.75
   d. $883.80

Answers

1. **b.** False. *The answer can be found in Section 3.1.*
2. **a.** True. *The answer can be found in Section 3.2.*
3. **b.** False. *The answer can be found in Section 3.3.*
4. **d.** $287.50. *The answer can be found in Section 3.1.*
5. **b.** $909.09. *The answer can be found in Section 3.2.*
6. **d.** $883.80. *The answer can be found in Section 3.3.*

Key Ideas

- The foregone interest, which could be earned if cash were received immediately, is called the opportunity cost of delaying its receipt.
- The time value of money and the mathematics associated with it provide important tools for comparing the relative values of cash flows received at different times.
- You can think of the present value as the amount that you have to put in the bank today to have some specific amount in the future.
- How frequently a loan's interest is compounded changes how we determine the time value of money.
- An amortized loan is a direct application of the present value of an annuity.

Critical Thinking Questions

1. Suppose you own some land, purchased by your father 20 years ago for $5,000. You are able to trade this land for a brand new Corvette sports car. What economic opportunity might you forego if you proceed with the trade? How would you estimate the opportunity cost of proceeding with the trade?
2. The Corvette dealership from Question 1 is also willing to trade the car for an IOU you own that promises to pay you $2,000 at the end of each year for the next 10 years and $20,000 when it matures at the end of the 10-year period. Investors are currently valuing such IOUs using a 6% discount rate. What economic opportunity might you lose if you make the trade? How would you calculate the opportunity cost of the trade?
3. If the market for new automobiles and the real estate and bond markets are all efficient, what do you think you would discover about the opportunity costs of the trade in Questions 1 and 2?
4. Usually we compute present values using a constant interest rate. But we know that interest rates vary over time, and it is impossible to know what the interest rate will be in 10 or 20 years. Why is using the current interest rate a good approach? Or would we be better off to simply ignore cash flows arriving beyond the period for which we have reasonable interest rate estimates? Explain your answer.
5. We discussed the EAR (effective annual percentage rate). Private student loans often are structured so no payments are necessary while a student is in school (and for 6 months after). However, interest does accrue during this period. This interest is then added to the principal amount of the loan once the grace period ends. For example, you borrow $10,000 at 6% when you start a two-year graduate program. The interest is $50 per month. You complete your degree and take advantage of some of the postgraduate grace period and then begin making payments 25 months after the loan began. The new principal is now $10,000 plus the *capitalized interest* of $1,250 or $11,250. Lenders don't state an effective annual rate because of the uncertainty associated with the amount of capitalized interest. Does this seem fair? Can you think of a way that this could be expressed so student borrowers understand what they are committing to when they get a private student loan with a capitalized interest feature?

Key Terms

amortized loan A loan that is paid off in equal periodic payments. Automobile loans and home mortgages are often amortized loans.

annuities due A finite stream of cash flows of a fixed amount, equally spaced in time where payments are made at the beginning of each period.

annuity A finite stream of cash flows of a fixed amount, equally spaced in time.

compounding Earning interest on previously earned interest.

discounting Solving for the present value of a future cash flow.

discount rate The interest rate used to find the present value of a future payment or series of payments. For many investments, investors' required return is the discount rate used to find the present value.

effective annual percentage rate (EAR) The annualized compound rate of interest.

future value A cash flow, or stream of cash flows, re-expressed as an equivalent amount at some future date.

interest The amount of money paid by a borrower to a tender for the use of the borrowed principal. The rate is expressed as a percentage of the principal owed.

nominal annual percentage rate (APR) The stated interest rate per year without considering the effect of compounding.

nominal rates The stated rate or yield that reflects expectations about inflation.

opportunity cost The amount of the highest valued forgone alternative.

ordinary annuity A finite stream of cash flows of a fixed amount, equally spaced in time, where payments are made at the end of each period.

periodic interest rate The rate of interest expressed per period, e.g., per month (12 periods per year); quarterly (4 periods per year); semi-annually (twice per year); weekly (52 periods per year); bi-annually (once every two years), etc.

perpetuities An infinite stream of equal cash flows, each equally spaced in time.

present value A future cash flow, or stream of cash flows, re-expressed as an equivalent current amount of money.

principal The amount of money borrowed.

simple interest The number of years multiplied by the interest rate multiplied by the amount originally invested.

time value of money The idea that, holding all else constant, people prefer to receive a given amount of money today rather than in the future.

Key Formulas

Future value of a single cash flow with compound interest

(3.4) $$FV_n = PV_0(1 + r)^n$$

Future value of a single cash flow with simple interest

(3.5) $$FV_n^s = PV_0 + (n)(PV_0)(r) = PV_0(1 + nr)$$

Future value of a single amount with continuous compounding

(3.6) $$FV_n = PV_0 e^{rn}$$

Present value of a single cash flow with compound interest

(3.7) $$PV_0 = FV_n(1 + r)^{-n} = FV_n \frac{1}{(1 + r)^n}$$

Present value of a single amount with continuous compounding

$$(3.8)\qquad PV_0 = FV_n(e^{-rn}) = FV_n\frac{1}{e^{rn}}$$

Effective Annual Percentage Rate

$$(3.9)\qquad EAR = \left(1 + \frac{APR}{CP}\right)^{cp} - 1$$

General formula for finding the present value of a cash flow stream

$$(3.10)\qquad PV_0 = \frac{CF_1}{(1 + r)^1} + \frac{CF_2}{(1 + r)^2} + \cdots + \frac{CF_N}{(1 + r)^N} = \sum_{n=1}^{N}\frac{CF_n}{(1 + r)^n}$$

General formula for finding the future value of a cash flow stream

$$(3.11)\qquad FV_N = CF_1(1 + r)^{N-1} + CF_2(1 + r)^{N-2} + \cdots + CF_N$$

Formula for the present value of a perpetuity

$$(3.12)\qquad PV_0 = \frac{CF}{r}$$

Present value of an ordinary annuity

$$(3.13)\qquad PV_0^{ord} = (CF)\left(\frac{1 - [1/(1 + r)^N]}{r}\right)$$

Present value of an annuity due

$$(3.14)\qquad PV_0^{due} = (CF)\left(\frac{1 - [1/(1 + r)^N]}{r}\right)(1 + r)$$

$$(3.15)\qquad PV_0^{due} = PV_0^{ord}(1 + r)$$

Future value of an ordinary annuity

$$(3.16)\qquad FV_N^{ord} = (CF)\frac{(1 + r)^N - 1}{r}$$

Future value of an annuity due

$$(3.17)\qquad FV_N^{due} = FV_N^{ord}(1 + r)$$

Web Resources

The importance of the time value of money concept is discussed here:
http://www.qfinance.com/cash-flow-management-calculations/time-value-of-money

The effective interest rate concept can be applied to mortgage interest rates, too. The Motley Fool website shows how to compute an effective after-tax interest rate here:
http://wiki.fool.com/How_to_Calculate_an_Effective_Mortgage_Rate_With_a_Tax_Writeoff

Mortgage agreements have a stated note rate, which determines the interest component of each payment, and an annual percentage rate (APR). The APR is almost always higher than the note rate because it includes other costs associated with acquiring a mortgage for a home: origination fee, points, prepaid interest, and insurance. Here is a description of the APR:
http://www.americanloansearch.com/info-apr.htm

For information on mortgage rates, mortgage calculators, and historic rate information, visit:
http://www.mortgagenewsdaily.com/mortgage_rates/